A NEW GEOMETRIC NOTATION FOR OPEN AND CLOSED-LOOP ROBOTS

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ABSTRACT

This paper presents a new geometric notation for the description of the kinematic of open-loop, tree and closed-loop structure robots. The method is derived from the well-known Denavit and Hartenberg (D-H) notation, which is powerful for serial robots but leads to ambiguities in the case of tree and closed-loop structure robots. The given method has all the advantages of D-H notation in the case of open-loop robots.

1. INTRODUCTION

Many methods are available for the description of the geometry of robots with open-chain mechanism [1]. The most common use is the elegant D-H method [2]. The D-H method is dealing with links with only two joints. The definition of a joint with respect to the preceeding one is carried out by means of 4 parameters. The use of D-H notation in robotics has facilited greatly all the modeling problems (geometric kinematics, and dynamics) [3]. The D-H notation, powerful and useful as it is, however, is still hampered by certain difficulties. In fact, the application of the D-H notation to robots with links having more than two joints is difficult and leads to ambiguities [4].

Sheth and Uicker (S-U) [4] has developed another notation which describes each link by 7 parameters. The S-U method can be used to describe any mechanism, but owing to its complexity it has been applied only for the closed-loop robots [5].

In this paper we propose a new geometric notation which can be used for both the closed and the open-loop robots. It has all the advantages of D-H notation when used for open-chain robots, and can easily be used for the closed-loop robots. In the case of links with 2 joints, 4 parameters are needed to describe a joint with respect to the preceeding one, while 2 additional parameters may be needed in the case of links with more than two joints.

In the following two sections we will present the D-H and the S-H notations. The proposed notation will be presented in section 4. Two examples will be given in section 5 to illustrate the given notation.

2. DENAVIT AND HARTENBERG NOTATION [1]

This method is the most popular in the robotics world. It can be used only in the case of serial robots. A robot is composed of n+1 links, link 0 is the fixed base, and link n is the terminal link, joint (i) connects links (i-1) and (i). A coordinate frame R_i is assigned fixed with respect to link (i). The axis of joint (i) is supposed along $\underline{Z}_{i-1}$ while the $\underline{X}_i$ axis is defined as the common perpendicular to $\underline{Z}_{i-1}$ and $\underline{Z}_i$ (Fig. 1-a).

The 4x4 transformation matrix which defines frame (i) with respect to frame (i-1) is obtained as function of 4 parameters $(\theta_i, r_i, d_i, \alpha_i)$ (Fig. 1a). This matrix denoted by ${}^{i-1}T_i$ is equal to :

$$
{}^{i-1}T_i = \mathrm{Rot}(Z,\theta_i)\ \mathrm{Trans}(Z,r_i)\ \mathrm{Trans}(X,d_i)\ \mathrm{Rot}(X,\alpha_i)
$$

$$
= \left|\begin{array}{c|c|c|c} \cos\theta_i & -\sin\theta_i \cos\alpha_i & \sin\theta_i \sin\alpha_i & d_i \cos\theta_i \\ \sin\theta_i & \cos\theta_i \cos\alpha_i & -\cos\theta_i \sin\alpha_i & d_i \sin\theta_i \\ \emptyset & \sin\alpha_i & \cos\alpha_i & r_i \\ \hline \emptyset & \emptyset & \emptyset & 1 \end{array}\right| \quad (1)
$$

If joint (i) is rotational, the joint variable q_i is equal to θ_i, while $q_i = r_i$ if joint (i) is prismatic. Hence $q_i = (1-\sigma_i)\,\theta_i + \sigma_i r_i$ where $\sigma_i = \emptyset$ if joint (i) is rotational and $\sigma_i = 1$ if joint (i) is prismatic.

The geometric model of a serial robot can thus be obtained by the successive multiplications of the transformation matrices :

$$
{}^{\circ}T_n = {}^{\circ}T_i\ {}^{i}T_2 \ldots {}^{n-1}T_n \quad (2)
$$

It is to be noted that the frame (n) can be always defined such that the D-H constant parameters of frame (n) are equal to zero.

Two remarks are to be given about the D-H notation :

i) The definition of the axis of joint (i) as $\underline{Z}_{i-1}$ is sometimes confusing, for this reason some people [6-7] find more convenient to define the axis of link (i) as $\underline{Z}_i$, but as a result of D-H notation the coordinate frame fixed with link (i) will be R_{i+1} (Fig. 1-b) which, in our opinion, is more confusing than the first case.

ii) It is impossible to use D-H notation as it is in the case of closed-loop structure, and not even in the case of tree structure. For example consider the situation shown in Fig. 2 which shows 3 rotational joints on a tree structure. Owing to D-H notation :

. R_o is defined such that $\underline{Z}_o$ is the axis of joint (1).

. Traversing from joint 1 to joint 2 will lead to define a coordinate frame R_1 fixed with respect to link (1), where $\underline{Z}_1$ is the axis of joint 2. The varia-

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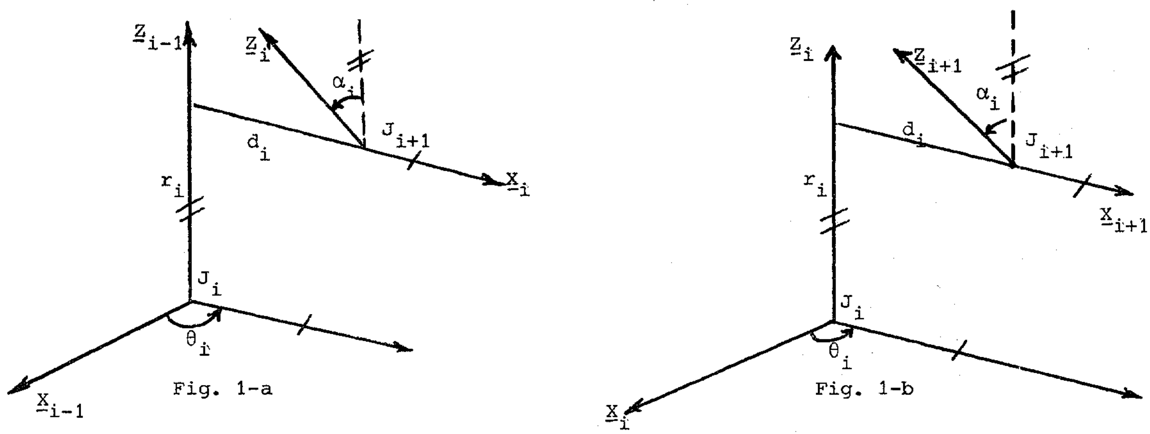

Figure 1. Denavit and Hartenberg Notation

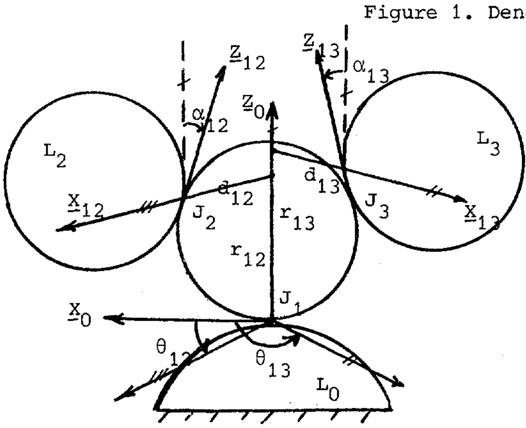

Figure 2. Ambiguities of D-H notation

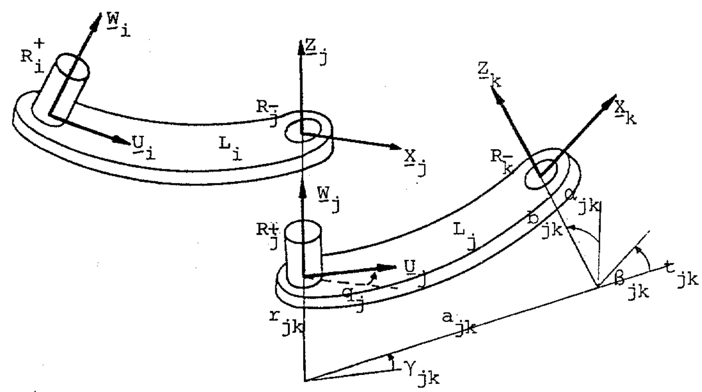

Figure 3. Sheth and Uicker parameters

ble of joint 1 is θ_1 and it is the angle between $\underline{X}_0$ and $\underline{X}_1$. α_1, d_1, r_1 can be defined as usual.

. Now traversing from joint 1 to joint 3 another frame is to be defined with $\underline{Z}$ is the joint 3 axis and is fixed also with respect to link (1). This frame is defined by some (θ,r,α,d) parameters but what subscripts do we have to assign for these parameters ?

A solution may proposed by the use of double subscripts such that when traversing from joint 1 to joint 2 the parameters will be denoted by $(\theta_{12}, r_{12}, \alpha_{12}, d_{12})$ and by $(\theta_{13}, r_{13}, \alpha_{13}, d_{13})$ when traversing from joint 1 to joint 3, the joint variable q_1 will be θ_{12} or θ_{13}, an additional constant parameter which specifies the relation between θ_{12} and θ_{13} is to be defined, i.e.

$$\theta_{12} = \theta_{13} + \gamma_{13} \tag{3}$$

Another confusion is still taking place because we have always two frames fixed with respect to link 1. How to call them R_{12} and R_{13} ? At any case the frame (i) is no more fixed with respect to link i. We see from that simple example that D-H notation will loose one of its best advantage which is its simplicity. And the mathematical formulas used in the robot modeling (geometric-kinematics and especially dynamics) will not be handy.

3. SHETH AND UICKER NOTATION [4]

Owing to the inefficiency of the D-H notation in representing the closed loop structure, Sheth and Uicker have developed another notation system, where each transformation matrix is composed of two parts :

i) a constant part specifying the shape of the link.

ii) a distinct variable part representing the joint motion.

Consider (Fig.3) which shows two successive links, each joint contains two coordinate systems. The first denoted $\underline{X}_j\ \underline{Y}_j\ \underline{Z}_j$ is an arbitrarily chosen system with $\underline{Z}_j$ is the joint axis, fixed with respect to link (j) and may be thought of as locating the position of the joint element R_j-. The other coordinate system $\underline{U}_j\ \underline{V}_j\ \underline{W}_j$ is also defined fixed in the mating joint element R_j+. It is chosen such that $\underline{W}_j$ lies along the joint axis $\underline{Z}_j$, but $\underline{U}_j$ and $\underline{V}_j$ are arbitrarily oriented.

The motion of the joint (j) is designated by q_j.

Now, it is necessary to define the parameters which describe the shape of the link and also the parameters which describe the joint motion.

3.1. Shape Matrix

The shape of each link (such as link (j) in Fig. 3 for example), is specified by the relative orientation between the coordinate system R_j+ at the "begining" of the link and R_k- at the "following" end. To determine the constant shape parameters for a link, the common perpendicular is found between the two axes $\underline{W}_j$ and $\underline{Z}_k$. This common perpendicular is assigned an arbitrary positive direction and is denoted as t_{jk}. Six parameters are required to define the shape of each link. They are defined for link (j) as shown in (Fig. 3) according to the following conventions :

a_{jk} = distance from $\underline{W}_j$ to $\underline{Z}_k$ measured about t_{jk}

α_{jk} = angle from positive $\underline{W}_j$ to positive $\underline{Z}_k$ measured about t_{jk}

b_{jk} = distance from t_{jk} to $\underline{X}_k$ measured about $\underline{Z}_k$

β_{jk} = angle from t_{jk} to positive $\underline{X}_k$ measured about $\underline{Z}_k$

r_{jk} = distance from $\underline{U}_j$ to t_{jk} measured along $\underline{W}_j$

γ_{jk} = angle from positive $\underline{U}_j$ to positive t_{jk} measured about $\underline{W}_j$

F_{jk} is the shape matrix of link (j) for the path from joint (j) to joint (k), its general form is :

$$
{}^{j^+}T_{k^-} = F_{jk}
$$

$$
\left|\begin{array}{c|c|c|c}
C\gamma_{jk}C\beta_{jk}-S\gamma_{jk}C\alpha_{jk}S\beta_{jk} & -C\gamma_{jk}S\beta_{jk}-S\gamma_{jk}C\alpha_{jk}C\beta_{jk} & S\gamma_{jk}S\alpha_{jk} & a_{jk}C\gamma_{jk}+b_{jk}S\gamma_{jk}S\alpha_{jk} \\
S\gamma_{jk}C\beta_{jk}+C\gamma_{jk}C\alpha_{jk}S\beta_{jk} & C\gamma_{jk}C\alpha_{jk}C\beta_{jk}-S\gamma_{jk}S\beta_{jk} & -C\gamma_{jk}S\alpha_{jk} & a_{jk}S\gamma_{jk}-b_{jk}C\gamma_{jk}S\alpha_{jk} \\
S\alpha_{jk}S\beta_{jk} & S\alpha_{jk}C\beta_{jk} & C\alpha_{jk} & r_{jk}+b_{jk}C\alpha_{jk} \\
\hline
\emptyset & \emptyset & \emptyset & 1
\end{array}\right| \qquad (4)
$$

3.2. Joint Matrix

The joint matrix will be denoted by $V_j(q_j)$ where q_j is the j^{th} joint variable. For a kinematic joint (j) having the coordinate systems R_j- and R_j+ attached to its preceeding and following elements respectively, the transformation between these coordinate systems is given by : $V_j(q_j)$. Two cases are to be considered :

Rotational joint : the joint variable q_j is given by the angle between $\underline{X}_j$ and $\underline{U}_j$ and is considered positive conterclockwise about positive $\underline{W}_j$.

Hence,

$$
{}^{j^-}T_{j^+} = V_j(q_j) = \left|\begin{array}{cccc}
\cos q_j & -\sin q_j & \emptyset & \emptyset \\
\sin q_j & \cos q_j & \emptyset & \emptyset \\
\emptyset & \emptyset & 1 & \emptyset \\
\hline
\emptyset & \emptyset & \emptyset & 1
\end{array}\right| \qquad (5)
$$

Prismatic joint : the joint variable q_j is the distance between $\underline{X}_j$ and $\underline{U}_j$ measured along $\underline{W}_j$, and is positive if in the direction of positive $\underline{Z}_j$.

Hence,

$$
{}^{j^-}T_{j^+} = V_j(q_j) = \left|\begin{array}{cccc}
1 & \emptyset & \emptyset & \emptyset \\
\emptyset & 1 & \emptyset & \emptyset \\
\emptyset & \emptyset & 1 & q_j \\
\hline
\emptyset & \emptyset & \emptyset & 1
\end{array}\right| \qquad (6)
$$

3.3. Transformation Matrix ${}^jT_k$

The transformation matrix ${}^jT_k$ between the coordinate systems R_j- and R_k- shown in (Fig. 3) is given by ${}^jT_k = V_j(q_j)\,F_{jk}$

The S-U notation has been used to study the closed-loop robots [5]. But as we see it is not convenient to use for a system where D-H can be used i.e. in the case of a serial robot. The complexity of this notation has been pointed out by Roth [8].

4. The Modified Notation

4.1. Introduction

The aim of the new notation is to define a method which can be used easily and without ambiguity in the closed-loop robots. We think that this aim has been fulfiled and we will show that the given notation can be used also for the open-loop robots as easy and general as that of D-H notation.

The proposed method defines the transformation matrix in the case of two-joint link by the use of 4 parameters as in the D-H notation. In the case of links with more than two joints two additional parameters may be needed.

The proposed notation is defined such that :

- The axis of joint (i) will be $\underline{Z}_i$
- The coordinate frame $R_i \triangleq (O_i, \underline{X}_i, \underline{Y}_i, \underline{Z}_i)$ is fixed with respect to link (i)
- The parameters which lead to define frame (i) will have (i) as subscript.

On the base of these assumptions we see that the ambiguities seen in section 2 can be avoided (Fig.4).

In order to find the parameters necessary to define the links frames we consider the following three cases : open-loop robots, tree-structure robots and closed-loop robots.

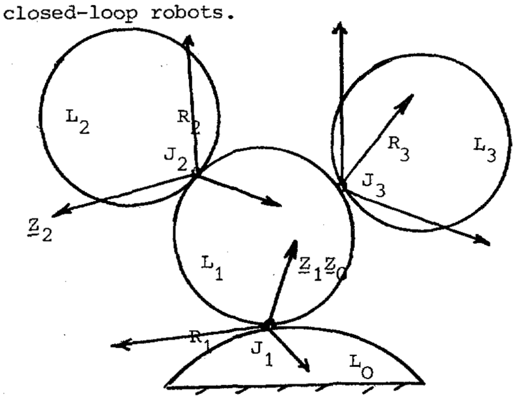

Figure 4. Principe of the new notation

4.2. Open-loop Robots

The system is composed of n+1 links, link (0) is the fixed base, while link (n) is the terminal link. Joint (i) connects link (i-1) and link (i).

Let :

R_i the fixed frame with respect to link (i)

$\underline{Z}_i$ the axis of joint (i)

$\underline{X}_i$ will be defined on the common perpendicular of $\underline{Z}_i$ and $\underline{Z}_{i+1}$ (Fig. 5).

The following parameters are required to define the frame (i) with respect to frame (i-1) : (Fig. 5)

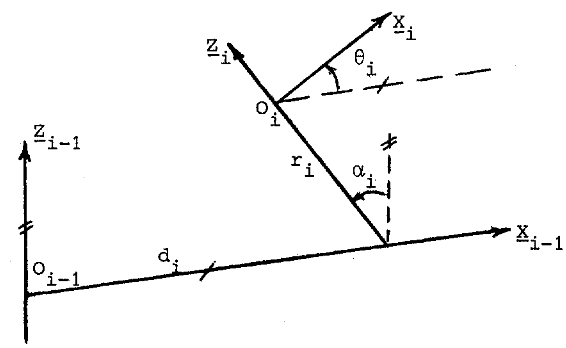

Figure 5. The new notation for a binary link

α_i angle between $\underline{Z}_{i-1}$ and $\underline{Z}_i$ about $\underline{X}_{i-1}$

d_i distance between 0_{i-1} and $\underline{Z}_i$

r_i distance between 0_i and $\underline{X}_{i-1}$

θ_i angle between $\underline{X}_{i-1}$ and $\underline{X}_i$ about $\underline{Z}_i$

The variable of joint (i) denoted by q_i is θ_i if (i) is rotational and r_i if (i) is prismatic. Hence

$q_i = \theta_i(1 - \sigma_i) + r_i\,\sigma_i$

where $\sigma_i = \emptyset$ if joint (i) is rotational and $\sigma_i = 1$ if joint (i) is prismatic.

The transformation matrix ${}^{i-1}T_i$ is equal to :

$$ {}^{i-1}T_i = \mathrm{Rot}(X,\alpha_i)\mathrm{Trans}(X,d_i)\mathrm{Rot}(Z,\theta_i)\mathrm{Trans}(Z,r_i) $$

$$ = \left|\begin{array}{ccc|c} \cos\theta_i & -\sin\theta_i & \emptyset & d_i \\ \cos\alpha_i\sin\theta_i & \cos\alpha_i\cos\theta_i & -\sin\alpha_i & -r_i\ \sin\alpha_i \\ \sin\alpha_i\sin\theta_i & \sin\alpha_i\cos\theta_i & \cos\alpha_i & r_i\ \cos\alpha_i \\ \hline \emptyset & \emptyset & \emptyset & 1 \end{array}\right| \quad (7) $$

It is to be noted that :

- R_o can be defined always such that $R_o \equiv R_1$ when $q_1 = \emptyset$ which means that : $\alpha_1 = d_1 = \bar{q}_1 = \emptyset$, where $\bar{q}_i = \theta_i\ \sigma_i + r_i(1 - \sigma_i)$
- $\underline{X}_n$ can be taken along $\underline{X}_{n-1}$ if $q_n = \emptyset$, which means that $\bar{q}_n = \emptyset$.

The proposed notation is similar to that of D-H. In fact it is to be noted that $(\theta_i, r_i, \alpha_{i+1}, d_{i+1})$ of the modified notation are respectively $(\theta_i, r_i, \alpha_i, d_i)$ of D-H.

It has been noted that this definition of the frame R_i fixed with link i such that $\underline{Z}_i$ is along the axis of joint (i), leads also to simplify the dynamic model of the robot [9].

The geometric model of the robot will be obtained by the successive multiplications of the transformation matrices :

$$ {}^{\circ}T_n = {}^{\circ}T_1\,{}^{1}T_2 \ldots {}^{n-1}T_n \quad (8) $$

4.3. Tree-Structure Robots

The system to be considered in this case is composed of n+1 links, n joints and m end effectors.

The links and the joints will be numbered as follows (Fig. 6)

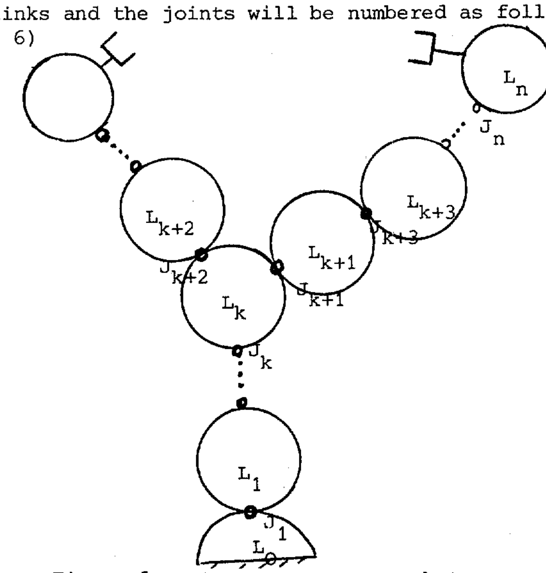

Figure 6. A tree-structure robot

- the base will be considered as link $\emptyset$
- the numbers of links and joints are increasing at each branch when traversing from the base to an end effector
- link (i) is articulated on joint (i), i.e. joint (i) connects the link (a(i)) and link (i), where a(i) is the number of the link antecedent to link (i) when coming from the base
- frame (i) is defined fixed with respect to link (i), and $\underline{Z}_i$ is the axis of joint (i).

In the case of a link with two joints the frame at the end of this link, say j, will be defined with respect to the frame at the begining of the link, say i = a(j), exactly as in the case of open-loop robot described previously, i.e by the aid of 4 parameters $(\alpha_j, d_j, \theta_j, r_j)$. $\underline{X}_i$ is the common perpendicular on $\underline{Z}_i$ and $\underline{Z}_j$.

In the case of links with more than two joints (Fig. 7), we define the different frames as follows

- find the common perpendiculars to $\underline{Z}_i$ and each of the succeeding axis, on the same link (i), $\underline{Z}_j$ where i = a(j) (j = k, ℓ, m, ...).

- let one of these common perpendiculars be the $\underline{X}_i$ axis, it is prefered to take $\underline{X}_i$ as that corresponding to the common perpendicular of the joint on which is articulated the longest branch, say k,

- the other perpendiculars will be denoted by $\underline{X}_i'$, X_i'',... Thus some other auxiliar frames $R_i'(O_i', \underline{X}_i', \underline{Y}_i', \underline{Z}_i)$ will be defined fixed with respect to link (i).

The matrix ${}^{i}T_k$ will be defined by the use of 4 parameters $(\alpha_k, d_k, r_k, \theta_k)$ as in Eq. (7). The other succeeding frames R_j ($j = \ell, m, \ldots$) will be defined in general by the following parameters (Fig. 7) :

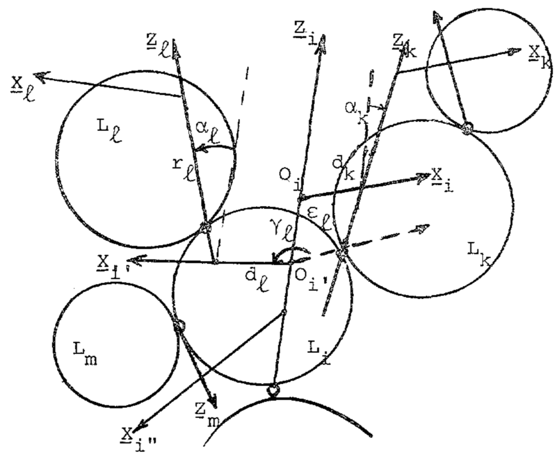

Figure 7. New parameters for a link with more than two joints

γ_j angle between $\underline{X}_i$ and $\underline{X}_i'$ about $\underline{Z}_i$

ε_j distance between O_i and O_i'

α_j angle between $\underline{Z}_i$ and $\underline{Z}_j$ about $\underline{X}_i'$

d_j distance between O_i' and $\underline{Z}_j$

θ_j angle between $\underline{X}_i'$ and $\underline{X}_j$ about $\underline{Z}_j$

r_j distance between O_j and X_i'

In this case ${}^{i}T_j$ will be defined by :

$${}^{i}T_j = {}^{i}T_{i'}\, {}^{i'}T_j$$

where

$$
{}^{i}T_{i'} = \mathrm{Rot}(Z,\gamma_j)\ \mathrm{Trans}(Z,\varepsilon_j) = \left|\begin{array}{cccc} \cos\gamma_j & -\sin\gamma_j & \emptyset & \emptyset \\ \sin\gamma_j & \cos\gamma_j & \emptyset & \emptyset \\ \emptyset & \emptyset & 1 & \varepsilon_j \\ \hdashline \emptyset & \emptyset & \emptyset & 1 \end{array}\right| \qquad (9)
$$

and ${}^{i'}T_j$ is identical to ${}^{i-1}T_i$ given in (Eq. 7). The parameters of ${}^{i'}T_j$ will have the subscript (j).

The matrix ${}^{i}T_{i'}$ is the unity matrix when both ε_j and γ_j are equal to zero.

It is to be noted that the frames R_i', R_i'',... are used only to define the new parameters when more than two joints are connected on the same link, but will not be used in developping the mathematical models where we directly use ${}^{i}T_j$.

The description of an end effector in the fixed frame R_o will be obtained by the successive multiplication of the matrices leading from the base to that end effector.

4.4. Closed-Loop Robots

The system in this case is composed of n+1 links, m end effectors, and ℓ joints. Thus the number of closed-loop is equal to $b=\ell-n$.

To get the transformation matrices, we define at first a tree-structure equivalent to the system by cutting each closed-loop through one of its joints. Many methods are available to show where the cutting process may take place [5,10,11]. For the purpose of this paper the cutting process can be assumed arbitrarily.

Once the equivalent tree-structure is defined the links and joints will be numbered as described in section 4.3, while the cut joints will be numbered from (n+1) to ℓ.

The links fixed frames of the equivalent tree structure will be defined as in section 4.3. Additional 2b frames will be defined fixed on the links which are connected by the cut joints. If (j) is the number of an opened joint, the corresponding additional frames defined on the links on both sides of that joint will be denoted by R_j and R_{j+b} (their $\underline{Z}$ axis is along the joint (j) axis) (Fig. 8).

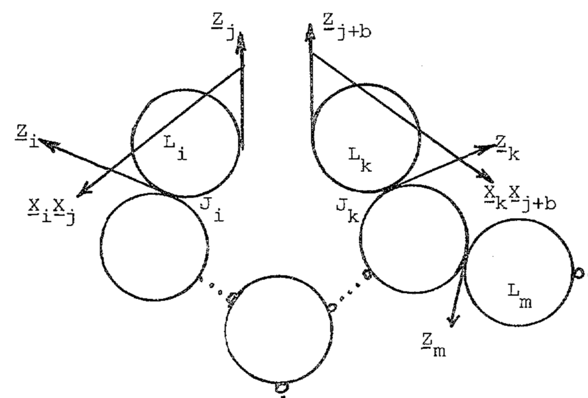

Figure 8. Cutting a loop at joint (j)

If the additional frame R_j is fixed on the link (i), and R_{j+b} is fixed on link (k), then R_j and R_{j+b} will be defined with respect to R_i and R_k as usual, and the parameters will get the subscript (j) and (j+b) respectively. These parameters are constants, furthermore $\theta_j, \theta_{j+b}, r_j, r_{j+b}$ can always be taken equal to zero by taking $\underline{X}_j = \underline{X}_i$ and $\underline{X}_{j+b} = \underline{X}_k$.

We can define the transformation matrix ${}^{j}T_{j+b}$ from one side of a cut joint to the other side, by the use of two parameters, one beeing the joint variable q_j. These parameters will be denoted by :

ρ_j = angle between $\underline{X}_j$ and $\underline{X}_{j+b}$ about $\underline{Z}_j$

τ_j = distance between O_j and O_{j+b} along $\underline{Z}_j$

The transformation matrix ${}^{j}T_{j+b}$ is defined by :

$$
{}^{j}T_{j+b} = \mathrm{Rot}(Z,\rho_j)\ \mathrm{Trans}(Z,\tau_j) = \left|\begin{array}{cccc} \cos\rho_j & -\sin\rho_j & \emptyset & \emptyset \\ \sin\rho_j & \cos\rho_j & \emptyset & \emptyset \\ \emptyset & \emptyset & 1 & \tau_j \\ \hdashline \emptyset & \emptyset & \emptyset & 1 \end{array}\right| \qquad (10)
$$

If joint (i) is rotational $\rho_j = q_j$ and if (i) is prismatic $\tau_j = q_j$.

The relations between the joints variables of a closed-loop will be obtained by expressing that the product of all the transformation matrices around the loop is equal to unity.

The transformation matrix between any two frames can be obtained by the multiplication of all the transformation matrices connecting these frames.

5. EXAMPLES

In this section we give the geometric parameters corresponding to a serial robot (Stanford) and a closed-loop robot (Hitachi HPR).

5.1. The geometric parameters of the Stanford manipulator, Fig. 9

This robot is a six degree of freedom robot, it has an open-loop structure. Applying the technique developed in section 4.1, we get the parameters of the robot as follows :

i	α	d	θ	r	σ
1	∅	∅	θ_1	∅	∅
2	-90	∅	θ_2	RL2	∅
3	90	∅	∅	r_3	1
4	0	∅	θ_4	∅	∅
5	-90	∅	θ_5	∅	∅
6	+90	∅	θ_6	∅	∅

5.2. The geometric parameters of Hitachi HPR Robot

This robot contains a single closed loop (Fig. 10) applying the new notation, the parameters of the robot are obtained as follows :"The loop has been opened at joint 8 between link 4 and link 5. ρ_8 is q_8 and τ_8 is equal to ∅."

a(i)	i	γ	ε	α	d	θ	r	σ
∅	1	∅	∅	∅	∅	θ_1	∅	∅
1	2	∅	∅	-90	∅	θ_2	∅	∅
1	3	∅	∅	-90	∅	θ_3	∅	∅
2	4	∅	∅	∅	d_4	θ_4	∅	∅
3	5	∅	∅	∅	d_5	θ_5	∅	∅
5	6	∅	∅	∅	d_6	θ_6	∅	∅
6	7	∅	∅	90	∅	θ_7	∅	∅
4	8	∅	∅	∅	$d_8=d_5$	∅	∅	∅
5	9	∅	∅	∅	$d_9=-d_4$	∅	∅	∅

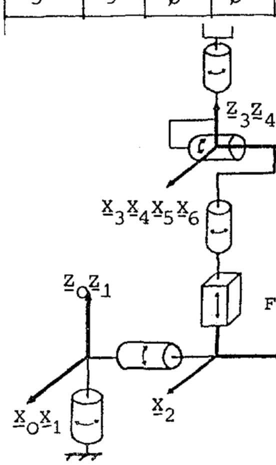

Fig.9 Stanford Manipulator

6. CONCLUSION

This paper presents a new notation which can be used to describe the open-loop robots and the closed-loop robots with a minimum of parameters and without ambiguities or difficulties. The method is derived from the popular D-H method.

The paper shows how to get the transformation matrices of any robotic mechanism by the use of the proposed method. A FORTRAN program has been developed to derive the symbolic transformation matrices automatically between any two frames of the system.

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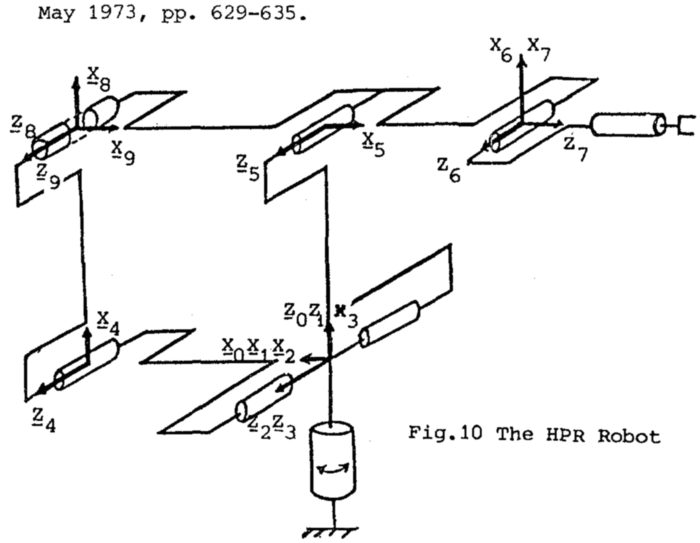

Fig.10 The HPR Robot